KINEMATIC MODEL CONSTRUCTION OF LOWER LIMB BASED ON LINKAGE MODEL

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Abstract

The human skeleton from mechanical point of view is a multi degree of freedom system supporting structure. Depending upon the fact whether the muscle groups are tightened, the supporting structure can be either determinate structure or statically indeterminate structure. The tightened muscles turn into the part of the supporting structure, the slack muscles make it possible that the structure operates as a mechanism. In this paper the simplified kinematics model of the lower limb and its analysis are presented.

Keywords: lower limb, kinematic model, biomechanical modeling

Introduction

The human lower limb made up of bones, joints and muscles is a multi degree of freedom system construction. The muscles yield a determinate construction in a tightened state. In this case all of the degrees of freedom of the construction are fastened by supporting. A slackened musculature limb is an unstable construction, acts as a mechanism, which moves due to the acting force and moment in the particular direction. Due to this construction which is unstable from mechanical point of view, a motion can be carried out. It can be achieved by tightening certain muscles and slacking others. Tightened muscles can be modelled by springs and act as rods in a construction.

The purpose of the paper is to subject the lower limb to kinematics analysis. On the base of fundamental mechanical theory I elaborated a very simple kinematics model of the lower limb. In the knowledge of basic principle, more sophisticated model can be constructed which takes more muscles and joints into account. It facilitates the more accurate modeling of operating of the lower limb. In the following chapters I outline the engineering fundamental concepts regarding the determinate and statically indeterminate structure, then I build up and analyze the kinematics model of the lower limb.


Table 1. Analysis of supporting

Degree of freedom: s
The number of the scalar data determining the position of the formation[1]:

	planar case	spatial case
mass point:	2 (x, y)	3 (x, y, z)
figure:	3 (x, y, φ_z)	6 (x, y, z, φ_x, φ_y, φ_z)

Determinate structure: s = n$_{supporting}$

s: number of the degree of freedom of the structure

n$_{supporting}$: number of the reactions of the structure

Unstable structure: s > n$_{supporting}$

Statically indeterminate structure:
   s < n$_{supporting}$

Analysis of jointed structures

The number of the equations: 3 × the number of the rods

The number of unknowns: the number of reactions of the whole structure: depending upon the supporting, plus the number of reactions between the rods: 2 × the number of pins[2]
   (Number of pins: number of the rods connecting with the pin, minus 1)

Application of the analysis in the case of structure containing 3 joints

E: the number of equations
U: the number of unknowns

$E = 2 \times 3 = 6$
$U = 2 + 2 + 2 = 6$
$E = U$, consequently all the reactions can be calculated

The equations of static state
– spatial: 6
– planar: 3 (generalized sum of F_x, F_y, M_z are 0), $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_i = 0$
For the whole structure:
   $M_a = 0 \Rightarrow Y_B \quad M_b = 0 \Rightarrow Y_A$

For the rod 1
$M_c = 0 \Rightarrow X_A$
$\sum X = 0 \Rightarrow X_{12}$
$\sum Y = 0 \Rightarrow X_{12}$

For the rod 2
$\sum X = 0 \Rightarrow X_B$

Steps of the kinematic model construction of the lower limb
– substitution of the shinbone and thighbone by rods
– substitution of the ankle, knee, hip joint by joint
– consideration of muscles as springs and damping
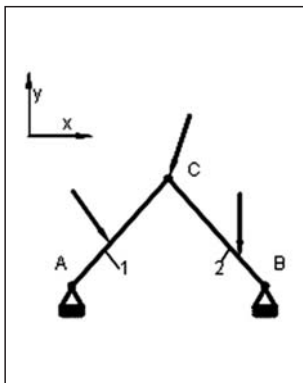– construction of the model as a mechanism

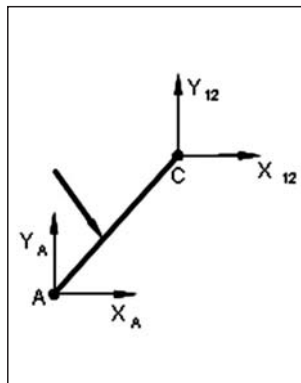*Figure 1.* Structure containing 3 joints
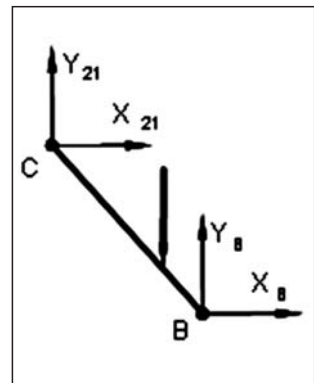

Figure 2. Analysis of rod 1

Figure 3. Analysis of rod 2

Approximate kinematic model
of the lower limb

Further simplifications
– substitution of the muscle groups only by
  one spring
– consideration of tightened springs as
  rods

Applied designations in the *Figure 4* and
Figure 5:
Bones:
   1-5, $\sum 5$
   1 – fibula + tibia
   2 – ossa metatarsalia
   3 – ossa tarsalia
   4 – femur
   5 – os coaxe

Muscles, sinew:
   $\alpha - \delta, \sum 4$
   α – Tendo calcaneus
   β – plantar muscles
   γ – musculus vastus
   δ – musculus gluteus
Joints connecting bones:
   a-f, $\sum 6$
   a – Chopart's joint
   b – ankle joint
   c – tarsal joint
   d – metatarsophlengeal joint
   e – knee joint
   f – hip joint
Joints fastening muscles to bone: g-n, $\sum 8$

Number of the rods (bones and muscles): 9
(It provides $9 \times 3 = 27$ mechanical equations)
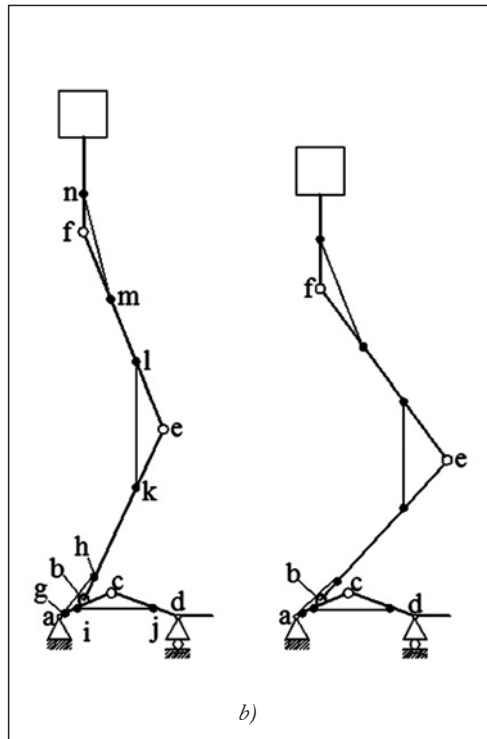

Figure 4. Build-up
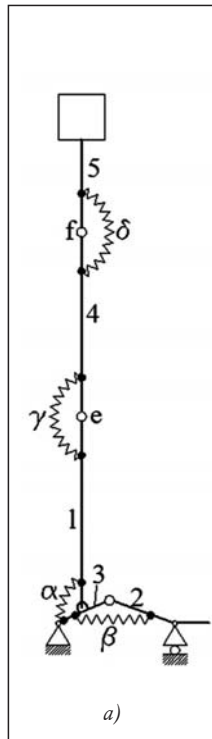and analysis
of the mechanism

a)

b)

Figure 5. Positions of the mechanism during the step
(a) muscles are flabby,
(b) muscles are tightened

Number of joints (between bones and between bone and muscle): $5+8 = 13$
(It provides $13 \times 2 = 26$ reaction forces)

Number of carrier idler set: 1
(It provides $1 \times 1 = 1$ reaction force)

Equations $= 9 \times 3 = 27$
Unknowns $= 13 \times 2 + 1 = 27$, consequently all the reactions can be calculated.

Conclusion

In possession of the expertise relating to the kinematics modeling of lower limb and limb, it becomes possible to design such artificial limbs which motion form is similar to the original one. The loads and forces acting on the skeleton made statically determinate by tightened muscles can be calculated, in this way the stressing of artificial limbs can be carried out.

References

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