THE ELEMENTS OF FLUID MECHANICS OF BILE FLOW THROUGH BILIARY DRAINAGE CATHETERS

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Abstract

Obstructive jaundice in the biliary tract can infect blood and result in mortality with a high rate. Percutaneous transhepatic biliary drainage (PTBD) with catheters is a useful solution discharging the obstructive jaundice. However, the elements of fluid mechanics showing clinical performance of a PTBD catheter have been documented little so far. In the article, empirical relationships between bile flow rate and pressure gradient in PTBD catheters were studied in terms of equivalent friction factor for the first time. Firstly, an equivalent friction factor in a catheter was raised and determined based on existing in vitro experimental data of bile flow through the catheters with different materials, various inner diameters and lengths under various pressure differences. Then, an empirical correlation of bile flow rate through a catheter was established based on pressure gradient, inner diameter and bile viscosity. The correlation was used to identify effects of catheter inner diameter and bile viscosity on the bile flow rate under the physiological bile pressure difference across obstructed common bile ducts. The feature of minor hydraulic losses in the catheters was clarified, too. The proposed equivalent friction factor was proportional to Reynolds number in a power of -0.654 in comparison with a power of -1 for the fully developed laminar flow in circular pipes. The bile flow rate through a catheter was proportional to inner diameter, kinematic viscosity, and pressure gradient in the powers of 3.2, -0.5 and 0.74, respectively. The minor hydraulic losses could be significant when Reynolds number was greater than 100.

Keywords: percutaneous transhepatic biliary drainage; biliary drainage catheter; bile; flow rate; friction factor; Reynolds number

Introduction

Obstructive jaundice is a special situation of jaundice when the bile is stopped flowing into the duodenum and remains in the blood due to gallstones in the common bile duct or extrinsic compression by tumours external to the common bile duct. The obstructive jaundice is a serious condition associated with high mortality rates and should be treated instantly by using percutaneous transhepatic biliary drainage (PTBD). PTBD is a procedure based on which the blocked bile is discharged into a drainage bag outside of the human body (external drainage in *Figure 1a*) or the duodenum through the common bile duct (internal drainage in *Figure 1b*) by using a catheter.[1,2] The internal drainage, the bile can flow in the internal catheter or flow into the bag thorough the external drainage catheter, depending on the resistance in the two catheters.

Even though PTBD is palliative, but it can improve quality of life for patients with benign and malignant biliary diseases with success rates of 82-99 %.[3-11] PTBD has been become an effective method for relief of biliary obstruction associated

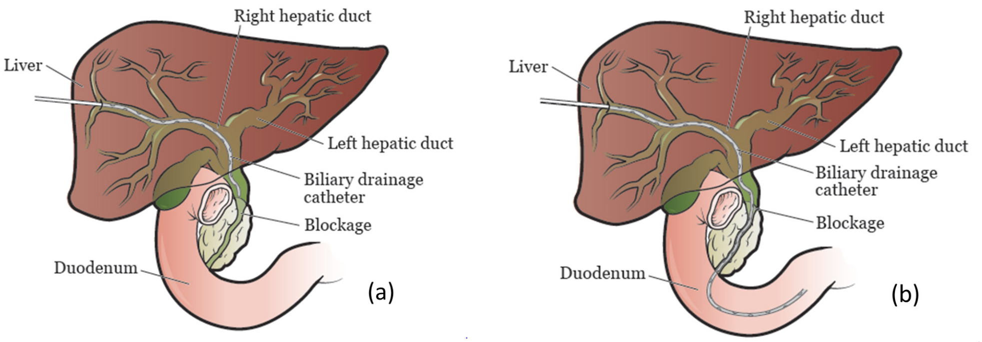

Figure 1. The external (a) and internal (b) percutaneous transhepatic biliary drainage, the pictures are from https://www.mskcc.org/cancer-care/patient-education/about-your-biliary-drainage-catheter

with both benign and malignant conditions.

Even though PTBD in patients is replaced every 4 or 6 weeks, unfortunately, PTBD is subject to complications after patients with malignant biliary disease undergo placement of drainage catheter.[6-8, 10, 12, 13] The complications can be cholangitis, catheter dislodgement, leaking around catheter, obstructed catheter, hemobilia, hypersecretion of bile, bilio-pleural fistula, bile duct perforation and pneumothorax[14], their occurrence percentages in 179 patients are illustrated in *Figure 2*. The total percentage of the complications related to catheter is as high as 43%. This means that the catheter performance plays a vitally important role in PTBD technique.

The catheter dislodgement and leaking complications connect with catheter design and soft tissue bimechanical property, while the catheter obstruction is associated with catheter design and bile fluid mechanics inside. Even though the catheter obstruction has made a 12.3% contribution to the total complication occurrence, it can cause catheter malfunction

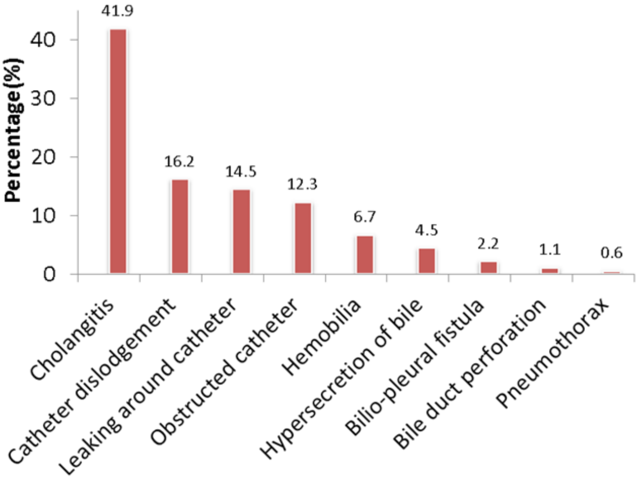

Figure 2. The percentages of complications after PTBD occurred in 179 patients with malignant biliary obstruction, the percentages were recalculated based on the data in the table in[14]

Material	D (French)	d (mm)	L (cm)	Flow rate Q(ml/s) under a pressure difference (cmH$_2$O)			
				1	2	6	9
Polyethylene	6.3	1.4	65	0.023	0.056	0.088	0.153
	7.1	1.5	65	0.028	0.068	0.110	0.165
	8.3	1.8	50	0.114	0.184	0.296	0.428
	10	2.2	50	0.210	0.385	0.495	0.701
Polyurethane	10	2.0	50	0.094	0.182	0.300	0.450
	12	2.7	50	0.256	0.456	0.769	1.110
Polyvinyl-chloride(PVC)	10	2.2	60	0.111	0.250	0.410	0.620
	10	2.2	50	0.153	0.333	0.500	0.800
Silicon elastomer	16	2.8	65	0.294	0.549	0.694	1.000
	16	2.8	50	0.310	0.595	0.833	1.330
Teflon	12	3.0	65	0.303	0.400	0.690	0.952
		3.0	31	1.300	1.540	2.000	2.500

Table 1. Measured flow rates through selected drainage catheters under specified pressure differences
Effective length of catheter   is the distance from catheter hub to first side hole, 1 French=1/3 mm

and eventually result in PTBD failure. From this point of view, attention should paid to design and fluid mechanics of catheter in PTBD.

Presently, investigations into design and fluid mechanis of catheter in PTBD are lack. Kerlan et al,[15] in the first time, measured bile flow rates through a series of different catheters at various pressure differences across the catheters in vitro, but their raw data remained unprocessed. Bret et al,[16] clinically applied large size silicone catheters with 12, 15, 18 French outer diameters (1 mm=3 French), 2, 3, 4 mm inner diameters and 3×5, 4×7, 5 mm×9 mm side holes into 30 patients with obstructive jaundice due to stenoses and tumours in bile ducts for a long-term. It was shown that PTBD was effective in treating benign and mailignant bile duct strictures for a long-term, but frequently minor problems, mostly catheter-related, did persist.[17]

A few in vitro experiments on fluid flow in percutaneous drainage catheters have been made by measuring drainage time at various catheter sizes and fluid viscosities.[18] It was demonstared that a more viscous fluid required more large catheter to secure a rapid drainage. The flow rates of commerical multipurpose pigtail drainage catheters were measured in vitro at 30 mmHg presssure difference. Since their inner diamter sizes were compariable, their flow rates were simliar in values.[19] The effects of number and location of drainage catheter side holes on liquid flow rate were measured in vitro by emplying unilateral and bilateral side hole models, the catheters with bilateal side holes had a higer flow rate than those with unilateral side holes, adding more side holes could not improve flow rate once the number of the holes beyond a critical number of holes.[20]

The flow rates of simulated bile such as water, three additional water solutions of guar gum (four dynamic viscosities) through three kinds of pigtail cathetes (two multipurpose drainage catheters, one bilary drainage catherter) were measured in vitro at 12 cmH$_2$O pressure difference under side hole unobstructed and

Catheter		**Bile**			
d (mm)	**L (cm)**	ρ **(kg/m3)**	μ **(Poise)**	ν **(mm^2/s)**	Δp **(cmH$_2$O)**
1.4	50	1000	0.01, 0.02	1, 2	11.8-18.4
1.8	50	1000	0.01, 0.02	1, 2	11.8-18.4
2.2	50	1000	0.01, 0.02	1, 2	11.8-18.4
2.7	50	1000	0.01, 0.02	1, 2	11.8-18.4

Table 2. The known parameters for a clinical application

obstructed conditions, and it was identified that the number of side holes did not affect in vitro biliary catheter drainage.[21] The influence of catheter connections of catheter drainage flow rate was identified in vitro experimentally, it was shown that flow rates could be decreased significantly by connections, especailly when the inner diamater of a connection was smaller than the inner diameter of the catheters connected.[22]

A method was proposed to predict the clogging effect drainage catheters based on in vivo rabbit experiments by monitoring intra-catheter pressure.[23] The commerical catheter for PTBD was ligated with a nylon thread just proximal to the first side hole to pervent the catheter obstruction caused from jejunobiliary reflux of the intestinal contents in internal PTBD.[24] Fracture of the PTBD catheter could occure and cause bile peritonitis.[25] Three new techniques were developed to retrieve fractured and intrahepatically dislodged PTBD catheters.[26]

Based on existing results on PTBD catheters mentioned above, bile fluid mechanics associated with clinical performance of catheters has been documented a little so far, and there are no empirical relationships for bile flow through biliary drainage catheters to assess their clinical performance in the literature currently. In the paper, the raw experimental data of a series of catheters on bile flow rate and pressure difference were analysed based on the elements of fluid mechanics to make the dead experimental data alive. An equivalent friction factor through the catheters was proposed and determined by using these observed data. An empirical relationship of bile flow rate through a catheter was estabilshed accordingly and applied to predict the bile flow rates through the catheters with various inner diameters under measured normal and abnormal biliary pressures and two bile viscosities in a common bile duct. The effects of catheter inner diameter and bile viscosity were clarified. The work can be meaningful to catheter fluid mechanics in biomedical/biomechanical engineering and clinical practice in PTBD procedure.

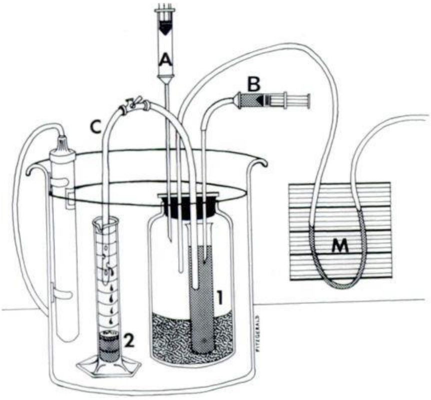

Figure 3. The experimental set-up for bile flow measurement through catheter in Kerlan et al.[15]
A bile stream is established in the catheter segment C by the pressure difference from chamber 1 to chamber 2 when the bile is at 37°C temperature

Experimental data

Catheters for PTBD are a flexile, plastic central-hollowed tube with taping head and side holes as shown *Figure 3a*. In vitro bile flow meansuremts through the catheters have been very rare in the literature so far. At the moment, just one relatively complete data set for such flow measurement was identified in Kerlan et al.[15] The experimental set-up in the mesurements is illustarted in *Figure 3b*. A catheter (C) connects chamber 1 and chamber 2. Chamber 1 is pressurised and depressurised by adding and removing air with syringe A to maintain a constant pressure difference across the catheter and estabilsh a bile flow in it. This pressure difference is measured by using U-tube manometer (M). Bile level is held to be contant with syringe A by infusing bile. All chambers are submerged in a water bath to allow bile temperautre to be at 37 °C.

Freshly aspirated human hepatic bile with a dynamic viscosity of 0.01 Poise (0.001 Pa·s) and a density of 1000 kg/m^3 serves as experimental fluid.[15] The experimental pressure differences, catheter sample lengths and diameters and measured bile flow rates are listed in *Table 1*.[15]

These raw exprimental data are going to be utilised to estabilish an empirical relationship between flow resistance factor/equivalent friction factor and Reynolds number and a correlation between bile flow rate and pressure gradient.

Empirical relationships

Equivalent friction factor

The hydraulic losses in the catheter shown in *Figure 3b* include the friction loss over the wet surface, the incidence loss at the inlet of catheter, the secondary flow loss in the 180° bend, and the diffusion loss across the side holes. At first, these losses are supposed to contribute an equivalent friction factor to simplify the problem; then, according to the skin friction factor formula for ducts,[27] the equivalent friction factor will be calculated based on a known pressure difference, bile flow rate, effective length of the catheter and inner diameter of the catheter in the following manner

$$\lambda = \frac{\Delta p}{\frac{V^2}{2g}\frac{L}{d}} \tag{1}$$

where is the gravity acceleration, $g = 9.81$ m/s^2, V is the mean bile flow velocity. Because of $V = Q/\left(\pi d^2/4\right)$[27], *Eq. (1)* can be rewritten as

$$\lambda = \frac{g\pi^2 d^5}{8Q^2}\frac{\Delta p}{L} \tag{2}$$

As a result, the corresponding λ - Re scattered data points based on the experimental data in *Table 1* is present in *Figure 4*, where

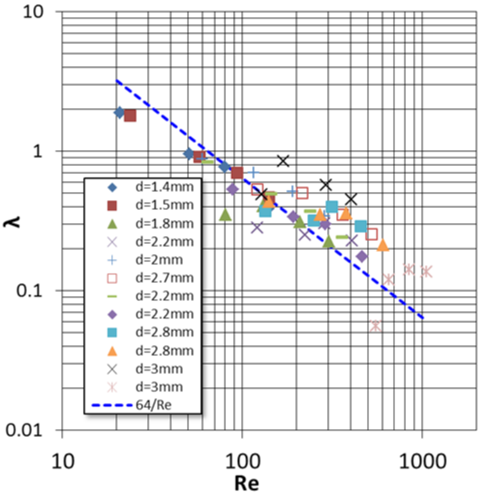

Figure 4. The scattered equivalent friction factors based on the experimental data in *Table 1* and the analytical friction factor are plotted as a function Reynolds number

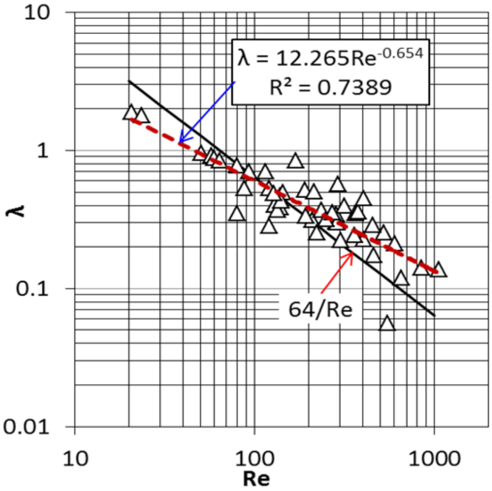

Figure 5. The fitted empirical correlation of equivalent friction factor in terms of Reynolds number and its comparison with the analytical friction factor for a catheter, the scattered data points are the same as those in *Figure 4*, the inner diameter is no longer indicated

$\mathrm{Re} = 4Q/\pi d\nu$, is the bile kinematic viscosity, $\nu = 1$ mm^2/s.[15] Because of $\mathrm{Re} \le 1000$, the bile flow in the experimental catheters was in laminar regime. For comparison, the analytical friction factor for the fully developed laminar flow in a circular pipe in[27] is plotted, too.

In the figure, some experimental points are below the $64/\mathrm{Re}$ curve as $\mathrm{Re} \le 200$, and some points are above the curve, suggesting the experimental shares a different slope with the analytical friction factor curve. The experimental has been best fitted by a power function of , the empirical formula is read as

$$\lambda = \frac{12.265}{\mathrm{Re}^{0.6540}}, \quad R^2 = 0.7389 \qquad (3)$$

where is the correlation coefficient. A comparison of the experimental with the fitted curve is made in *Figure 5*.

The relationship of bile flow rate to pressure gradient

An empirical correlation for λ has been established by *Eq. (3)* based on fluid mechanics method, the λ expression is involved into *Eq. (2)*, and following equation is achieved

$$\frac{12.265}{\mathrm{Re}^{0.6540}} = \frac{g\pi^2 d^5}{8Q^2}\frac{\Delta p}{L} \qquad (4)$$

Putting the Reynolds number $\mathrm{Re} = 4Q/\pi d\nu$ into *Eq. (4)*, and the following equation is established

$$\frac{12.265}{\left(\dfrac{4Q}{\pi d\nu}\right)^{0.6540}} = \frac{g\pi^2 d^5}{8Q^2}\left(\frac{\Delta p}{L}\right) \qquad (5)$$

From *Eq. (5)*, the bile flow rate through a catheter with the length driven by a pressure gradient can be solved and expressed by

$$Q = \frac{8.3186 g^{0.7429} \pi d^{3.2288}}{128\nu^{0.4859}}\left(\frac{\Delta p}{L}\right)^{0.74} \qquad (6)$$

where the units of Q, d, L, Δp and ν are m^3/s, m, m, mH2O, kg/m^3, m^2/s, respectively. In the equation, $Q \propto d^{3.2}\, \nu^{-0.49}\, (\Delta p/L)^{0.74}$ is held approximately.

For the fully developed laminar flow in a catheter, the friction factor is expressed analytically by $\lambda = 64/\mathrm{Re}$.[27] Involving the Reynolds number $\mathrm{Re} = 4Q/\pi d\nu$ into *Eq. (2)*, an analytical relationship between bile flow rate and pressure gradient is worked out

$$\frac{12.265}{\mathrm{Re}^{0.6540}} = \frac{g\pi^2 d^5}{8Q^2}\frac{\Delta p}{L} \qquad (7)$$

where the dynamic viscosity μ is related to the kinematic viscosity with $\mu = \rho\nu$, ρ is the bile density, the unit of Δp is mH$_2$O. In these expressions, $Q \propto d^4\ \nu^{-1}\left(\Delta p/L\right)^1$, suggesting parameters d, ν and $\Delta p/L$ exhibit a stronger effect on Q in comparison with those in *Eq. (6)* originated from experimental observations.

Another sort of relation between bile flow rate and pressure gradient

In the relationship of bile flow rate to pressure gradient section, an empirical friction factor is determined first, then it replaces the λ in *Eq. (2)*, and Re is experienced in terms of bile rate; finally, a relationship between bile flow rate and pressure gradient is sought as expressed with *Eq. (6)*. In fact, we can derive an empirical relationship between bile flow rate and pressure gradient directly based on analytical *Eq. (7)*. This method has used in the determination of an empirical expression between bile flow rate and pressure gradient across animal biliary tree in vitro in.[28] This method will be tried on the experimental data on the catheters[15] here.

A few perfusion experiments were performed on the biliary tree (hepatic, cystic and common bile ducts) of six fasting mongrel dogs[28] by using saline and bovine bile at different bile perfusion flow rates, respectively. The pressure differences across the tree were recorded. It was identified that the experimental scattered points of ($\Delta p/L$,) and ($\rho g\Delta p/L$, $128\mu Q/\pi d^4$) can be best fitted with a linear relationship in a log-log plot. Then the bile flow rate can be obtained in terms of pressure gradient, like *Eq. (7)*.

In doing so, the experimental data points of $\Delta p/L$ - $128\nu Q/\pi g d^4$ and $\rho g\Delta p/L$ -are plotted and fitted, respectively, for the experimental data in the catheters in [15]. The experimental scattered data points and the corresponding regression formulas are illustrated in *Figure 6*. Based on these formulas, the bile flow rate through a catheter under a known pressure gradient is given by

$$Q = \frac{0.3234\pi g d^4}{128\nu}\left(\frac{\Delta p}{L}\right)^{0.6458} \qquad \text{or}$$

$$Q = \frac{8.573\pi d^4}{128\mu}\left(\frac{\rho g\Delta p}{L}\right)^{0.6458}, R^2 = 0.7487 \quad (8)$$

where the units of Q, d, L, Δp, ρ, ν and μ are m3/s, m, m, mH2O, kg/m3, m2/s, Pa.s, respectively.

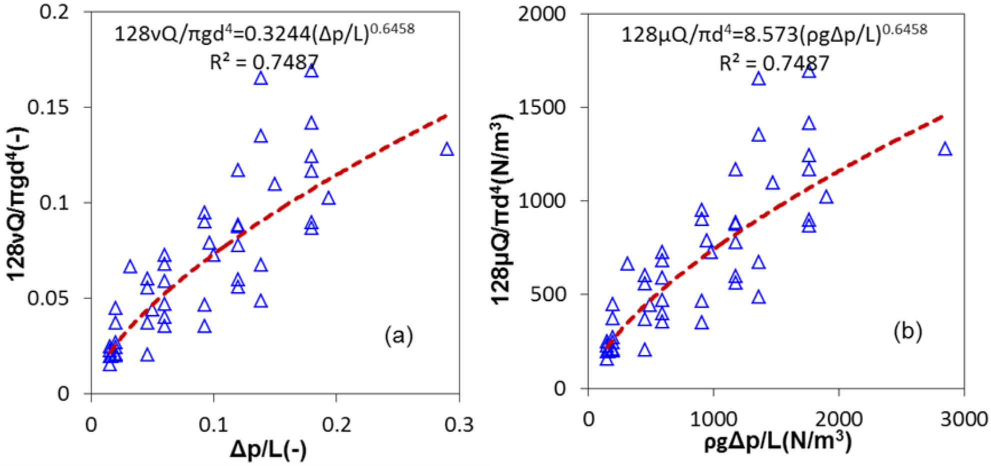

Figure 6. The scattered data points of ($\Delta p/L$, $128\nu Q/\pi g d^4$) and ($\rho g\Delta p/L$, $128\mu Q/\pi d^4$) as well as the corresponding regression formulas, the scattered data points are the same as those in *Figure 4*, the inner diameter is no longer indicated

A comparison of two kinds of relationship

From the same set of experimental data, two relationships have been obtained for bile flow rate in terms of pressure gradient across a catheter expressed by *Eq. (6)* and *(8)*. Two relationships may result in a different bile flow rate under the same clinical condition. To confirm this effect, a computational example is provided here.

The biliary mean resting pressure in normal human common bile duct is 11.8 cmH$_2$O, but in the duct with obstructive jaundice, it is 18.4 cmH$_2$O.[29,30] It is assumed that a 50 cm long catheter is connected to a common bile duct with obstructive jaundice at the 18.4 cmH$_2$O initial pressure, after drainage persists for a certain long of time, the biliary pressure restores to the normal level of 11.8 cmH$_2$O. This means that the pressure difference across the

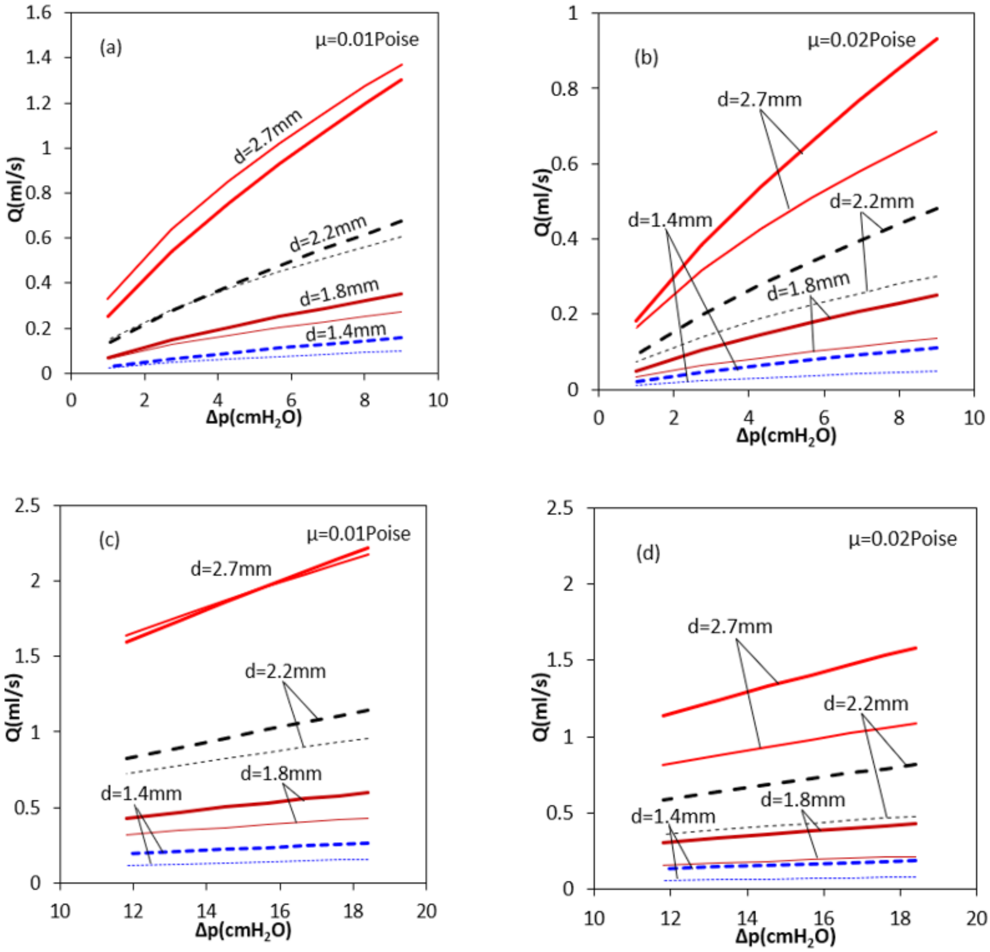

Figure 7. The predicted bile flow rate through four catheters in terms of bile pressure difference across the catheters at two viscosities, the thick lines are for *Eq. (6)*, but the thin lines for *Eq. (8)*; in *(a)* and *(b)*, the pressure difference is the range of the experiments in *Table 1*; while in *(c)* and *(d)*, the pressure difference is based on clinical observation[29, 30]

catheter varies to 11.8 cmH_2O from 18.4 cmH_2O. The bile dynamic viscosity is 0.01 Poise 15 and 0.02 Poise 31 with a density of 1000 kg/m3. The catheter inner diameters are $d = 1.4$, 1.8, 2.2 and 2.7 mm, respectively, based on *Table 1*. These known parameters are summarised in *Table 2*.

Firstly, *Eq. (6) and (8)* are used to predict the bile flow rates under the experimental conditions such as 1, 2, 6 9 cmH_2O pressure differences and 0.01 Poise viscosity as shown in *Table 1* and at four inner diameters in *Table 2*. The two equations result in nearly the same bile flow rate profiles as shown in *Figure 7a*. This is not surprised because they have originated from the same experimental data set and applied under nearly the same condition in terms of pressure difference, viscosity and catheter inner diameter as in the experiments.[15]

Secondly, two equations are employed to estimate the bile flow rates at 0.02 Poise viscosity, while the rest condition remain the same as those for *Figure 7a*. In the experiments of[15] the tested liquid viscosity was kept being 0.01 Poise. The prediction at 0.02 Poise viscosity is an extrapolation from the results at 0.01 Poise viscosity. The flow rates from Eq. (6) are larger than those from *Eq. (8)*, as demonstrated in Figure 7b because of $Q \propto \nu^{-0.49}$ in *Eq.(6)* rather than $Q \propto \nu^{-1}$ in *Eq. (8)*. These suggest that the flow rates predicted with two equations at a viscosity more than 0.01 Poise are not accurate as those at 0.01 Poise.

Finally, two equations are utilized to calculate $Q - \Delta p$ curves at four inner diameters and two viscosities in *Table 2* and under the pressure differences higher than those in *Table 1*. The predicted $Q - \Delta p$ curves are illustrated in *Figure 7c* and *d*. These predictions are extrapolation from an experimental pressure difference in[15] to a higher-pressure difference in clinical observation. Once again two equations lead to a very similar flow rate curve at to 0.01 Poise viscosity, but a very different curve at 0.02 Poise viscosity.

Clearly, the bile flow rate rises with both increasing pressure difference and inner diameter but reduces with increasing viscosity. The effect of inner diameter on the flow rate is the most significant in comparison with that of the other factors. To secure a relatively high bile flow rate and better drainage, a catheter should prefer an inner diameter as big as possible, especially for thick bile.

From *Figure 7c* and *d*, since the bile flow rate is inversely proportional to the viscosity in *Eq. (8)*, the viscosity in *Eq. (8)* exhibits a stronger effect on the flow rate than the viscosity does in *Eq. (6)*. As a result, the flow rates predicted with *Eq. (8)* are smaller than those with *Eq. (6)* in most cases. In the case of $d = 2.7$ mm and $\mu = 0.01$ Poise, two equations result in nearly the same flow rate. This is because of the dominated effect of inner diameter on the flow rate.

In the experiments in [15], the fluid viscosity was kept constant. Thus, there is no effect of fluid viscosity reflected both *Eq. (6)* and *(8)*. If the viscosity varied in the experiments in,[15] *Eq. (6)* and *(8)* should lead to a nearly identical bile flow under the same clinical condition. To validate two empirical relationships of *Eq. (6)* and *(8)*, more experimental data on in vitro bile flow measurements in catheters are desirable with more viscous liquids and under pressure differences higher than 9 cmH_2O.

Discussions

In the paper, a set of raw experimental data on in vitro bile flow through a series of catheter samples in[15] was analysed in fluid mechanics context. An empirical equivalent friction factor was figured out and the corresponding flow rate formula of bile through a catheter was

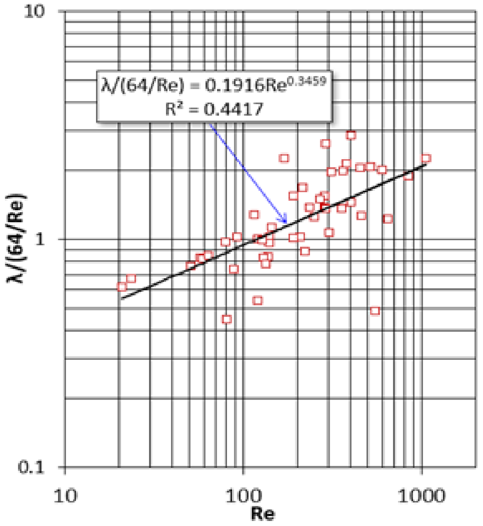

Figure 8. The ratio of the equivalent friction factor to the theoretical friction factor of the fully developed laminar flow in a circular pipe, i.e. as a function of Reynolds number, the scattered data points are the same as those in *Figure 4*, the inner diameter is no longer indicated.

established. Finally, an application example was demonstrated and the effects of catheter inner diameter, pressure difference and bile viscosity were clarified. Such a study has not been found in literature so far. The study in the paper has contributed to the elements of fluid mechanics of catheter in PTBD.

In the equivalent friction factor, there are minor hydraulic losses, namely entry loss at the catheter inlet, secondary flow loss in the bend of a catheter, and diffusion loss through the side holes in the catheter. It is not easily to measure and estimate these minor losses. Here using the ratio of the equivalent friction factor to the theoretical friction factor for the fully developed laminar flow in a circular pipe, i.e. $\lambda/(64/\mathrm{Re})$ is used to estimate these minor losses. As a result, the scattered data points and a regression equation are illustrated in *Figure 8*. Clearly, ratio augments with increasing Re, particularly, if Re > 100, then $\lambda/(64/\mathrm{Re}) > 1$, indicating the dominant minor losses. When Re < 100, the ratio is less than one. This effect may be due to some errors in the experiments or the thickening effect of non-Newtonian bile at low flow rate.

Note that, in clinical practice, the bile flows into the side holes of a catheter rather than out of the holes as shown in the experiments as shown in *Figure 3*. The diffusion loss in two scenarios may be different each other. This issue needs to be confirmed experimentally in the future. CFD studies on the minor hydraulic losses in biliary drainage catheters are also worthy of being attempted.

Recently, the flow rates in three commerical multipurpose pigtail drainage catheters at 30 mmHg presssure difference were measured in vitro with water by Macha, Thomas and Nelson in 2006.[19] The flow rates of water, three water solutions of guar gum across three pigtail

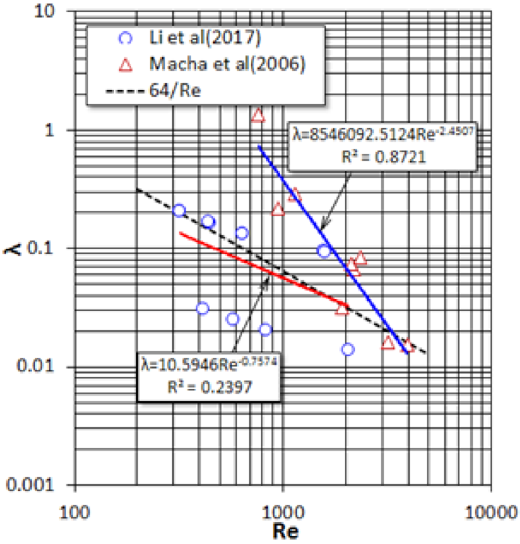

Figure 9. The experimental equivalent friction factors respectively by Macha, Thomas and Nelson[19] and Li, Ballard and D'Agostino[21] and fitted empirical correction of in terms of Reynolds number and its comparison with the analytical friction factor for a catheter.

cathetes (two multipurpose drainage catheters, one biliary drainage catherter) were in vitro measured at 12 cmH_2O pressure difference under side hole unobstructed and obstructed conditions, and it was identified that the number of side holes don't affect in vitro bilary catheter drainage.[21] The catheter geometrical parameters were presented in [19] and [21]. The flow rates in *Figure 4* in [19] and *Figure 2* in [21] for the unobstructed catheters were read and the equivalent frictiom factors were calculated by them with *Eq. (2)*, and the results are illustrated in *Figure 9*.

Clearly, the data points in two experiments are quite few and the Reynolds number is in the range of 300-4000, which is higher than that (20-1000) in *Figure 4*. The factors from the experimental data in[19] exhibit siginifcant variation. Even though the resgrssion equation for them is sightly below the analytical curve of $64/\mathrm{Re}$, its correlation coefficent is as small as 0.24.

The friction factors from the experimental data in [21] are considerably higher than the analytical curve as $\mathrm{Re} \le 2000$. Nonetheless further experimental comfirmation is on demand.

Since there is one viscosity in the experiments and no information about the used bile rheology in [15], the bile in fluid mechanics model is considered Newtonian. Ooi et al measured the bile dynamic viscosity and found that the bile rheology of 20 out of 59 patients is Newtonian.[32] Reinhart, Naf and Werth found the bile of the majority samples from the common bile duct of 138 patients (64.5 %) are Newtonian.[33] These facts suggest that the Newtonian bile model seems to be reasonable. In some cases, however, the bile can be non-Newtonian,[32-35] therefore the correlation needs be updated in the future based on in vitro experimental data on non-Newtonian fluid flow through PTBD catheters.

The bile viscosity can vary significantly across patients, for example, the dynamic viscosity of gallbladder bile is 0.0177-0.08 Poise[32], and even higher in gallbladder bile of patients with cholesterol (0.05 Poise) and mixed stones (0.035 Poise) compared to hepatic bile (0.02 Poise) [31]. Therefore, more in vitro studies on bile flow through a catheter with a variety of viscosities need to be launched in the future.

Conculisons

Based on a set of existing in vitro bile flow measurements through the catheters made with five kinds of material and in various inner diameters and lengths for PTBD application under different pressure differences across the catheters, an equivalent friction factor was put forwarded and determined. Furthermore, an empirical correlation of bile flow rate through a catheter to pressure gradient, inner diameter and bile viscosity was developed and applied to clarify effects of variable catheter inner diameter and bile viscosity under the physiological bile pressure differences in obstructed common bile ducts. The effect of minor hydraulic losses in the catheters was identified. It was shown that the proposed equivalent friction factor was proportional to Reynolds number in a power of -0.654 rather than -1 for the fully developed limner flow in circular pipes. The bile flow rate through a catheter was proportional to inner diameter, kinematic viscosity, and pressure gradient in the powers of 3.2, -0.5 and 0.74, respectively. The minor hydraulic losses could be dominant as Reynolds number was higher than 100. Further work includes in vitro bile flow measurements at different viscosities, non-Newtonian bile effect on bile flow rate through a catheter and minor hydraulic losses estimation in biliary drainage catheters based CFD simulations.

Compliance with Ethical Standards

Funding: This study was not funded by any organizations.
Conflict of interest: The author has no conflict of interest.
Ethical approval: This article does not contain any studies with human participants or animals performed by the author.

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